Improvements on a Green's Function Method for the Solution of Linearized Unsteady Potential Flows

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This paper introduces a lifting surface formulation and the functional variable linking concept in Morino's method. These new capabilities, possibly combined with a multipole expansion, greatly enhance the efficiency of the method when it is used in the aeroelastic analysis of complex configurations. Numerical examples are shown to demonstrate that these concepts can be used profitably in unsteady airload calculations.

Introduction

THE analysis of flutter and dynamic aeroelastic responses of complete and complex configurations remains a formidable task even if it is assumed that a linearized aeroelastic formulation is acceptable. This is mainly because of the extensive aerodynamic analyses that must be carried out for each configuration of interest in order to obtain a complete set of generalized unsteady aerodynamic forces for all the displacement modes needed to correctly model the aeroelastic behavior and for a set of reduced frequencies and Mach numbers sufficient to allow their correct interpolation for any flight condition. Nevertheless, thanks to the computational power available today, these analyses are becoming a routine matter for the aeroelastician, especially because of the widespread availability of computational tools capable of determining generalized unsteady aerodynamic forces for entire configurations.[1-3]

Rather recently, the analysis of Ref. 4, generally known as Morino's method, has developed beyond the experimental phase to reach the maturity of a production version[5] and results are now available that demonstrate its wide capabilities compared to other, sometimes more efficient, but less general methods.[6] The advantage of Morino's method over others used for complex configurations is the adoption of an approach that allows the calculation of unsteady forces for arbitrary reduced complex frequencies (i.e., not only purely harmonic), together with the capability of correctly modeling the aircraft without the excessive geometric simplifications implied in other formulations.[2,3] The interest in the approach, for both steady and unsteady flows, is witnessed by many other software developments that have been independently undertaken on the basis of the same formulation.[7-9]

The wide acceptance and maturity of the method are also demonstrated by the search for improved and more efficient numerical approximations[10] that can make it more usable in routine aeroelastic calculations. This paper introduces further enhancements aimed at an improvement of the numerical efficiency of Morino's method when it is used in practical aeroelastic analyses. These enhancements will allow the calculation of lifting loads without the need to evaluate both the upper and lower pressure distributions of the lifting surface itself and the attainment of higher-order elements on the basis of an already available zero-order formulation. Furthermore, it is also shown how the application of multipole approximations can positively affect the performances of Morino's method. Since these improvements greatly reduce the calculations needed to compute the influence coefficients, their number, and the number of unknowns required for results of comparable precision, it is thought that an order-of-magnitude reduction in cost can be obtained easily when Morino's method is used in practical aeroelastic design.

Short Summary of Morino's Method and Notation

Morino's method is an unsteady time domain or complex frequency domain formulation based upon Green's function method applied to the equation of linearized velocity potential for both subsonic and supersonic flows. The method is well documented,[5] so the presentation here will be limited to a very short summary introducing the notations and concepts needed to explain the new developments outlined in this paper. The presentation will be limited to subsonic flows for the sake of conciseness and because the use of the method in supersonic flow calculations is, at this time, limited. Nevertheless, the techniques presented here are applicable generally and not limited to subsonic flows.

Applying a Prandtl-Glauert transformation to the linearized perturbation potential equation and approximating the surface of the aircraft and its wake with quadrilateral panels on which the unknown potential is constant, we obtain a linear system of algebraic equations of the type,[5]

$$[Y]\{\varphi\} = [Z]\{\psi\} \tag{1}$$

where $\{\varphi\}$ and $\{\psi\}$ are the Laplace transform of the unknown potentials and downwashes at the center of each panel. The aerodynamic influence coefficients, i.e., matrices $[Y]$ and $[Z]$, are given by

$$Y_{jh} = \delta_{jh} - (C_{jh} + pD_{jh})e^{-p\theta_{jh}} - \sum_n (F_{jn} + pG_{jn})e^{-p(\theta_{jn}+\tau_n)}S_{nh} \tag{2a}$$

$$Z_{jh} = B_{jh}e^{-p\theta_{jh}} \tag{2b}$$

The summation term in Eq. (2a) is extended to the panels of the wake. S_{nh} is zero for all panels not having a direct connection to the wake; its value is ± 1 for the trailing-edge panels in order to ensure that the potential discontinuity for all the wake panels is equal to the convected difference of the potential of the corresponding upper and lower trailing-edge panels. Furthermore, p is the complex reduced frequency, i.e., $p = s\ell/U_\infty$, with s being the complex circular frequency, ℓ a reference

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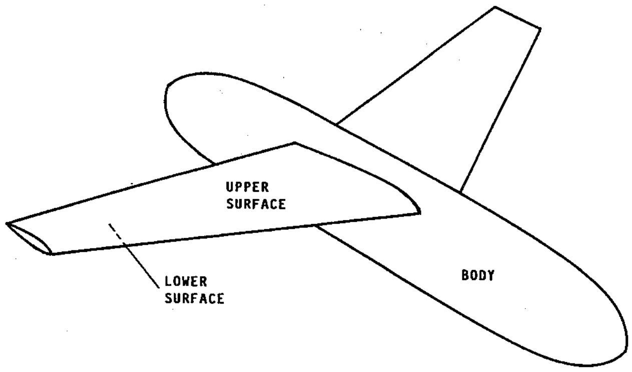

Fig. 1 Notations for the lifting surface approximation in a complete configuration.

length, and U_∞ the velocity of the undisturbed flow. The other terms in Eqs. (2) are given by[5]

$$B_{jh} = -\frac{1}{2\pi}\iint_{\Sigma_h} \frac{1}{R} \mathrm{d}\Sigma_h \tag{3a}$$

$$C_{jh} = -\frac{1}{2\pi}\iint_{\Sigma_h} \frac{1}{R^2}\frac{\partial R}{\partial N} \mathrm{d}\Sigma_h \tag{3b}$$

$$D_{jh} = -\frac{1}{2\pi}\iint_{\Sigma_h} \frac{1}{R}\frac{\partial \hat{\theta}}{\partial N} \mathrm{d}\Sigma_h \tag{3c}$$

$$F_{jn} = -\frac{1}{2\pi}\iint_{\Sigma_n'} \frac{1}{R^2}\frac{\partial R}{\partial N_u} \mathrm{d}\Sigma_n' \tag{3d}$$

$$G_{jn} = -\frac{1}{2\pi}\iint_{\Sigma_n'} \frac{1}{R}\frac{\partial \hat{\theta}}{\partial N_u} \mathrm{d}\Sigma_n' \tag{3e}$$

$$\theta_{jh} = [M(X_h - X_j) + |\bar{P}_h - \bar{P}_j|]M/\beta \tag{3f}$$

$$\theta_{jn} = [M(X_n^{(w)} - X_j) + |\bar{P}_n^{(w)} - \bar{P}_j|]M/\beta \tag{3g}$$

$$\pi_n = \beta[X_n^{(w)} - X_{h(n)}] \tag{3h}$$

From the above presentation, it clearly appears that, in order to improve the efficiency of the method, it is important to reduce both the cost of calculating the coefficients of Eqs. (2) and the number of unknowns without sacrificing the accuracy of the results. It must be remarked that only the incremental loading distributions related to structural motions and/or gusts are required in linearized aeroelastic analyses of lifting surfaces. Thus, for these components it is not necessary to calculate the actual pressure distribution in the many repeated calculations at different reduced frequencies and Mach numbers, as the loadings related to thickness, camber, yaw, and angle of attack are constant for each Mach number and can be calculated once for all.

This fact is very important since it implies a halving of the unknowns needed to represent a lifting surface and, in fact, the need to completely model the lifting surfaces in load calculations has been considered a limitation of Morino's method[11] and has suggested the hybrid approach of Ref. 1. We will now see how this limitation can be withdrawn from Morino's approach, thus greatly improving this method's use in aeroelastic analyses. Also, this approach does not require the re-evaluation of the coefficients of Eqs. (3) at different reduced frequencies, but only the assembly of Eqs. (2) and the solution of the resulting algebraic system, [Eq. (1)].

Lifting Surface Approximation in Complete Configurations

Limiting, for the sake of simplicity, our presentation to a body with a single lifting surface as sketched in Fig. 1, we can partition Eq. (1) as

$$[Y_{UU}]\{_U\} + [Y_{UL}]\{_L\} + [Y_{UB}]\{_B\} = [Z_{UU}]\{\psi_U\} + [Z_{UL}]\{\psi_L\} + [Z_{UB}]\{\psi_B\} \tag{4a}$$

$$[Y_{LU}]\{\varphi_U\} + [Y_{LL}]\{\varphi_L\} + [Y_{LB}]\{\varphi_B\} = [Z_{LU}]\{\psi_U\} + [Z_{LL}]\{\psi_L\} + [Z_{LB}]\{\psi_B\} \tag{4b}$$

$$[Y_{BU}]\{\varphi_U\} + [Y_{BL}]\{\varphi_L\} + [Y_{BB}]\{\varphi_B\} = [Z_{BU}]\{\psi_U\} + [Z_{BL}]\{\psi_L\} + [Z_{BB}]\{\psi_B\} \tag{4c}$$

where the subscripts *B, U,* and *L* refer to the body, upper, and lower surfaces of the lifting surface. Defining

$$\{\varphi_U\} = \{\varphi_M\} + \{\Delta\varphi\}$$
$$\{\varphi_L\} = \{\varphi_M\} - \{\Delta\varphi\}$$
$$\{\psi_U\} = \{\psi_M\} + \{\Delta\psi\}$$
$$\{\psi_L\} = \{\psi_M\} - \{\Delta\psi\} \tag{5a}$$

with

$$\{\varphi_M\} = (\{\varphi_U\} + \{\varphi_L\})/2$$
$$\{\Delta_\varphi\} = (\{\varphi_U\} - \{\varphi_L\})/2$$
$$\{\psi_M\} = (\{\psi_U\} + \{\psi_L\})/2$$
$$\{\Delta_\psi\} = (\{\psi_U\} - \{\psi_L\})/2 \tag{5b}$$

and then replacing Eq. (4a) with the sum of Eqs. (4a) and (4b), Eq. (4b) with the difference of Eqs. (4a) and (4b), and introducing the expressions of Eqs. (5) into Eqs. (4), we can write

$$([Y_{UU}] + [Y_{UL}] + [Y_{LU}] + [Y_{LL}])\{\varphi_M\} + ([Y_{UU}] - [Y_{UL}] + [Y_{LU}] - [Y_{LL}])\{\Delta\varphi\} + ([Y_{UB}] + [Y_{LB}])\{\varphi_B\} = ([Z_{UU}] + [Z_{UL}] + [Z_{LU}] + [Z_{LL}])\{\psi_M\} + ([Z_{UU}] - [Z_{UL}] + [Z_{LU}] - [Z_{LL}])\{\Delta\psi\} + ([Z_{UB}] + [Z_{LB}])\{\psi_B\} \tag{6a}$$

$$([Y_{UU}] + [Y_{UL}] - [Y_{LU}] - [Y_{LL}])\{\varphi_M\} + ([Y_{UU}] - [Y_{UL}] - [Y_{LU}] + [Y_{LL}])\{\Delta\varphi\} + ([Y_{UB}] - [Y_{LB}])\{\varphi_B\} = ([Z_{UU}] + [Z_{UL}] - [Z_{LU}] - [Z_{LL}])\{\psi_M\} + ([Z_{UU}] - [Z_{UL}] - [Z_{LU}] + [Z_{LL}])\{\Delta\psi\} + ([Z_{UB}] - [Z_{LB}])\{\psi_B\} \tag{6b}$$

$$([Y_{BU}] + [Y_{BL}])\{\varphi_M\} + ([Y_{BU}] - [Y_{BL}])\{\Delta\varphi\} + [Y_{BB}]\{\varphi_B\} = ([Z_{BU}] + [Z_{BL}])\{\psi_M\} + ([Z_{BU}] - [Z_{BL}])\{\Delta\psi\} + [Z_{BB}]\{\psi_B\} \tag{6c}$$

In view of the need to evaluate only the loading of the lifting surface, we can assume it to be symmetric in thickness and thin to the extent that the upper and lower surfaces are practically coincident within an acceptable numerical difference that has to be defined. With these hypotheses, we can write

$$([Y_{UU}]+[Y_{UL}]-[Y_{LU}]-[Y_{LL}])=0 \tag{7a}$$

$$([Y_{UU}]-[Y_{UL}]-[Y_{LU}]+[Y_{LL}])=2([Y_{UU}]-[Y_{UL}]) \tag{7b}$$

$$([Z_{UU}]+[Z_{UL}]-[Z_{LU}]-[Z_{LL}])=0 \tag{7c}$$

$$([Z_{UU}]-[Z_{UL}]-[Z_{LU}]+[Z_{LL}])=2([Z_{UU}]-[Z_{UL}]) \tag{7d}$$

because of the symmetry of the surface profile, and

$$([Y_{BU}]+[Y_{BL}])=\mathcal{O}(0) \tag{7e}$$

because the wing is very thin. Moreover, keeping in mind that in aeroelastic analyses we are considering only incremental loadings,

$$\{\psi_M\}=\mathcal{O} \tag{7f}$$

Using Eqs. (7), having obtained $\{\varphi_M\}$ from Eq. (6a), and discarding terms of $\mathcal{O}(0)$, we can write Eqs. (6b) and (6c) as

$$2([Y_{UU}]-[Y_{UL}])\{\Delta\varphi\}+([Y_{UB}]-[Y_{LB}])\{\varphi_B\}$$
$$=2([Z_{UU}]-[Z_{UL}])\{\Delta\psi\}+([Z_{UB}]-[Z_{LB}])\{\psi_B\} \tag{8a}$$

$$([Y_{BU}]-[Y_{BL}])\{\Delta\varphi\}+[Y_{BB}]\{\varphi_B\}$$
$$=([Z_{BU}]-[Z_{BL}])\{\Delta\psi\}+[Z_{BB}]\{\psi_B\} \tag{8b}$$

In this way, we limit the number of unknowns to only $\{\varphi_B\}$ and $\{\Delta\varphi\}$, from which the loads on the body and lifting surface can be directly calculated. It is pointed out that in the case of an isolated wing, Eq. (8a) introduces an exact equivalent lifting surface in Morino's method, as it approximates a lifting surface as a layer of doublets in almost the same way as in Refs. 1 and 11, except that the boundary condition here does not directly imply the imposition of null normal velocity on the surface of the aircraft.

The advantages implied by Eqs. (8) are: 1) a great reduction of the number of influence coefficients that must be evaluated, especially in the case of a fine idealization of lifting surfaces [compare Eq. (8) with Eq. (4)]; 2) a halving of the unknowns related to lifting surfaces; and 3) a simplification of the geometric modeling of lifting surfaces. The last point is due to the fact that, with this technique, a lifting surface can be approximated by parallel plates with a wake emanating from the middle of its trailing-edge panels. The distance between these plates can be of the order of 1/1000 of the chord with results that have been experimentally found to remain unchanged from 1/40 downward. This is true if a 36 bit machine is adopted, but it is very likely that they will remain almost unchanged for a 32 bit computer, as can be inferred from other results available in the literature.[12]

Clearly the concept just explained can be extended to any lifting surface of complete and complex configurations including tails, pods, and/or nacelles; in these cases, it will improve the efficiency of the method even more dramatically. Moreover, it can be implemented in a way that allows the user to decide what part of an aircraft can be influenced by another part. In this way, the user can decide not to calculate the aerodynamic coefficients of a part or section having only negligible effect. Also, note that with respect to the method of Ref. 1, which is believed to be one of the best methods for calculating unsteady subsonic harmonic loadings, Morino's method now requires almost the same number of unknowns; however, it has the advantage of not needing a recalculation of the aerodynamic coefficients at changing frequencies, but only the assembling implied by Eqs. (2).

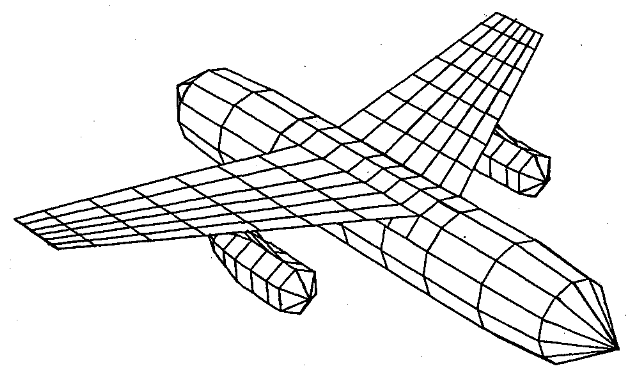

Fig. 2 Test configuration used to illustrate the lifting surface approximation.

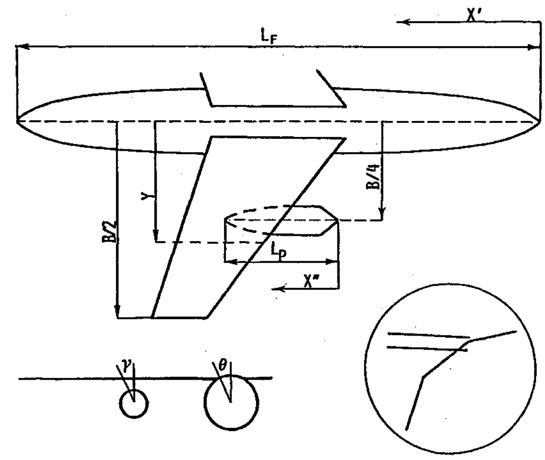

Fig. 3 Illustration of terms used in the presentation of the results and detail of flat-plate approximation of lifting surfaces.

The results obtained by the approximation just introduced are now presented for the test configuration of Fig. 2, which has a large body and a wing, with pylon and nacelle, placed rather distantly from the centerline. This geometry and the use of large panels for the wing are purposely designed to amplify the interference effects with the consequent magnification of the discarded error terms in the previously described formulation. It must be remarked that this test is aimed only at proving the validity of the approximation introduced in this paper and not the efficiency and accuracy of Morino's method, which is well documented in the literature.[4,6,7,13,14] Figure 3 sketches the flat-panel approximation and the symbols needed to interpret the results of a test calculation carried out for plunging and pitching motions at Mach $=0.8$ and $p=0.5j$. The results displayed in Figs. 4 and 5 clearly demonstrate that the lifting surface approximation, which is exact for the loadings of isolated wings, correctly fits the results obtained by using the original Morino method in the analysis of a complex configuration with strong interference effects.

When compared with the complete analysis, the approximate calculation required 25% fewer unknowns and saved 20% of computing time. This saving would clearly be more substantial if a finer mesh were used for the lifting surfaces. The derivatives of the potential needed to obtain the wing loadings are evaluated with a method that will be described in the next section.

Functional Variable Linking and Derivative Evaluation

A further reduction of the unknowns can be obtained for a very detailed zero-order approximation and it can also be combined with the concept outlined in the previous section. The method is herein called functional variable linking and is an idea that is already known to aerodynamic analysts[15]; but, it appears to have been seldom used, probably because of the difficulty in adopting it to complex configurations. As used

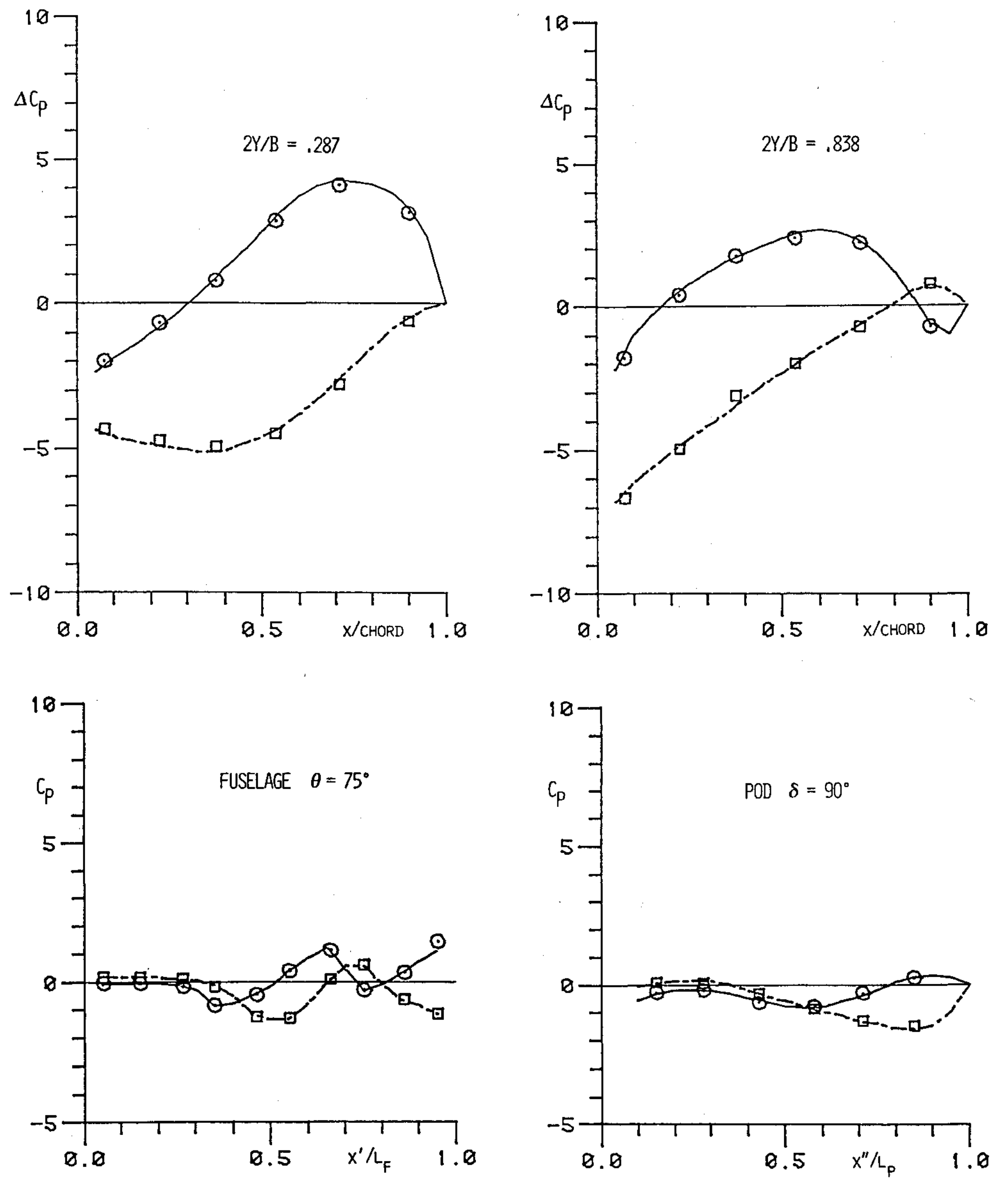

Fig. 4 Comparison of results for a plunging mode at Mach = 0.8, $p=0.5j$: Complete analysis: real —, imag ---; simplified analysis: real ○, imag □.

here, the method consists of applying to Eq. (1) the following transformation:

$$\{\varphi\} = [\tau]\{\tilde{\varphi}\} \tag{9}$$

where $\{\tilde{\varphi}\}$ has a number of components, say m, that is far less than that of $\{\varphi\}$, say n, thus obtaining the overdetermined system of equations,

$$[Y][\tau]\{\tilde{\varphi}\} = [Z]\{\psi\} \tag{10}$$

The matrix $[\tau]$ is sparse and defined by means of a discretization that superimposes a new set of finite-element-like patches onto those used to approximate the $\{\varphi\}$ vector with a zero-order approximation. Each of these new patches generally comprise many zero-order panels.

Taking the vector $\{\tilde{\varphi}\}$ as the potential values at the nodes of high-order patches, Eq. (9) allows a generalized functional linking of the zero-order approximation to the higher-order one. Figure 6 shows a possible scheme for linking a group of zero-order unknowns to a two-dimensional parabolic approximation. Here, we assume that the i panel lays within the k patch and that its center is defined by two local coordinates $\bar{\xi}$ and $\bar{\eta}$. Then we can write

$$\varphi_i = \sum_{ie}^{g} \tau_{(k)e}(\bar{\xi},\bar{\eta})\tilde{\varphi}_{(k)e} \tag{11}$$

where $\tau_{(k)e}$ is in this case defined by appropriate Lagrangian shape functions.[16] The preceding equation defines the nonzero elements of row i in matrix $[\tau]$, while the nonzero columns of the same row clearly depend on the position of the nodal component $\tilde{\varphi}_{(k)e}$ in vector $\{\tilde{\varphi}\}$. Obviously, other kinds of linking can be used and, for nonflat surfaces, ξ and η must be a suitable set of surface coordinates. In this way, the matrix $[\tau]$ is very sparse, requires relatively few calculations for its application, and can be easily generated for any configuration. The drawback of this approach is the need to generate two meshes, one for evaluating the integrals of Eqs. (3) and one for the functional linking. Nevertheless, it must be noted that the zero-order elements can be generated automatically from the functionally linked elements and that this approach is equivalent to the use of higher-order elements with an approximate integration of the coefficients of Eqs. (3).

As already pointed out, the system of equations obtained after the application of the transformation of Eq. (9) has more equations than unknowns and it is solved by a least square approach, as is often done in the solution of aerodynamic integral equations by collocation methods.[17] In Fig. 8, we show the results obtained by applying the variable linking to a fully flapped lifting surface with an oscillating flap,[18] according to the scheme of Fig. 7. In this way, the system unknowns are reduced from the original 400 to just 112. The lifting surface is modeled with parallel plates at a distance of 1/100 of the chord and the approximation of the previous paragraph,

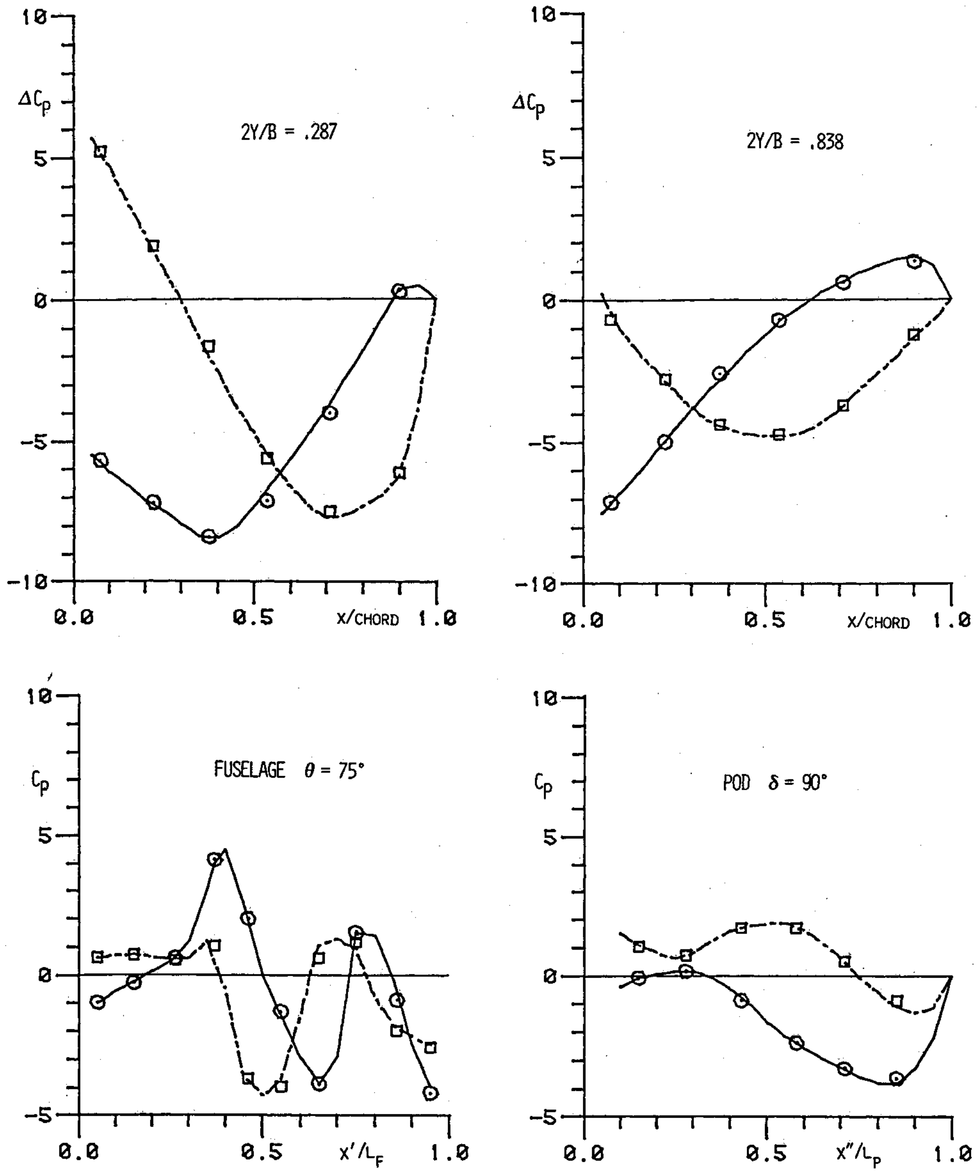

Fig. 5 Comparison of results for a pitching mode at Mach = 0.8, $p=0.5j$: Complete analysis: real —, imag ---; simplified analysis: real ○, imag □.

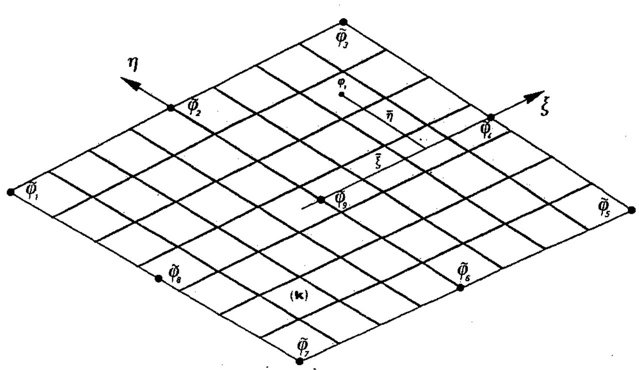

Fig. 6 A possible scheme of variable linking.

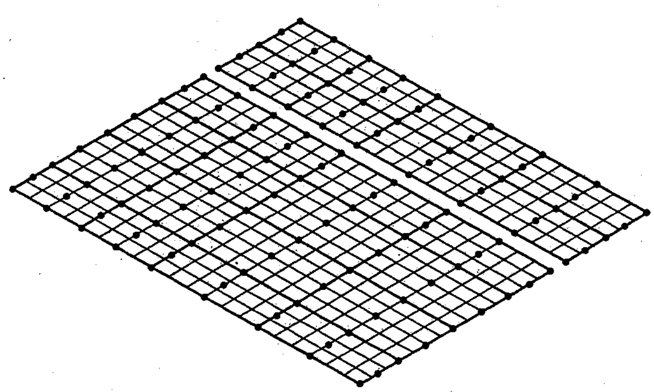

Fig. 7 Variable linking adopted for a fully flapped wing.

which is exact in this case, is applied so that the linking is used only for the potential difference in order to derive just the pressure loading. The loading distribution obtained by the direct solution of the full system of 400 linear equations related to the zero-order approximation is practically coincident with the classical result of Ref. 18. In Fig. 8, it is compared with that provided by variable linking to demonstrate the usefulness of the approach, especially if one notes that both leading-edge and control surface singularities are correctly and precisely interpolated without any a priori definition of the singularity type.

The concept of variable linking can also be used to directly evaluate the potential derivatives after the results of the zero-order approximation have been obtained. In this case, a linking is applied a posteriori with a quadratic patch overlapping the zero-order panels to approximate both the surface geometry and the potential distribution (see Fig. 9); this approximation is used to evaluate the pressure distributions at

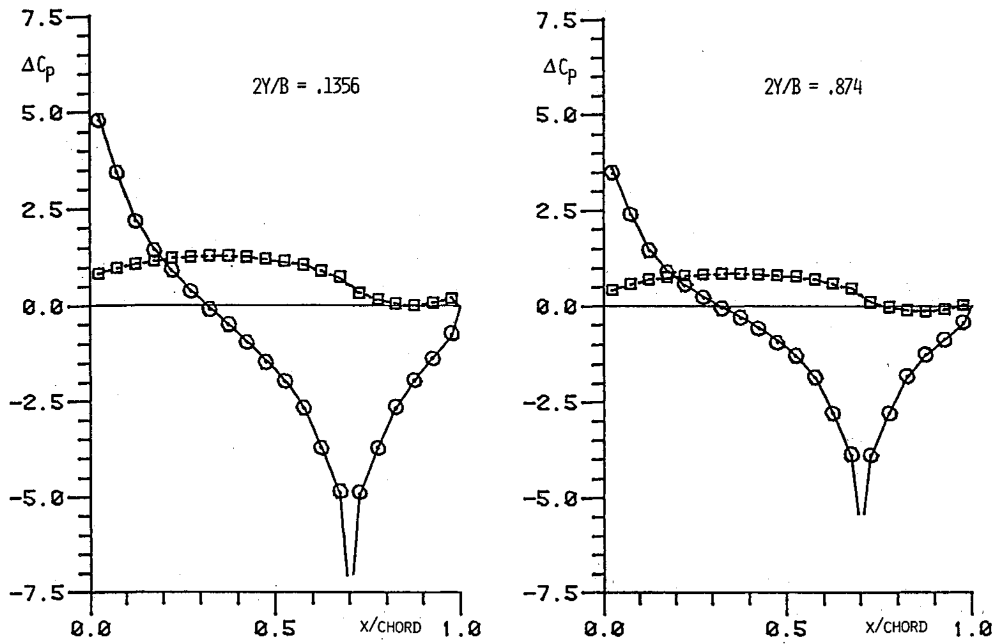

Fig. 8 Comparison of results for the wing of Fig. 7 oscillating in pitch and with oscillating flap. Complete analysis: real —, imag ---; functional variable linking: real ○, imag □.

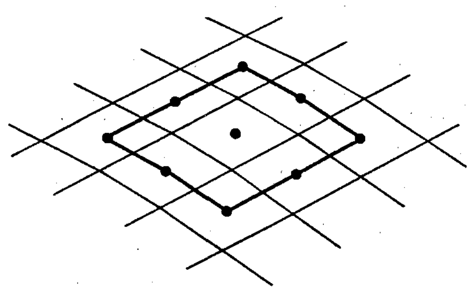

Fig. 9 Quadratic patch for the calculation of potential derivatives.

the center node. The method thus becomes a generalization of the central-difference approximation to surface derivatives, except at surface singularities and in places of high curvature where it must be used for external nodes to generalize forward or backward differences with a consequent reduction of precision. Even if the variable linking allows a direct evaluation of the derivatives, in the case of a coarse linking, the recovery of the zero-order potentials and the successive calculation of their derivatives by means of the above approximation can give slightly better results than those obtained directly from the shape functions used for the linking.

In our experience, this approximation of the derivatives of the potential has always given superior or at least equal results with respect to other techniques used in connection with Morino's method,[7,9,19] with the advantage of affording pressure coefficients that can always be related to the center of the panel in a consistent way. The reduction in computer cost is primarily related to the ratio of the operations needed for the direct solution of a complex system of equations and the operations needed to calculate the coefficients of the least square system, because the cost of applying the transformation of Eq. (9) and the solution of the real symmetric least square system is generally negligible. It can be seen that this ratio is of the order of $3/2\ (m/n)^2$ and is confirmed in this case where the time necessary for the solution of the reduced system has been about one-fifth of that required for the zero-order solution. Note also that this reduction is related to the lifting surface approximation introduced in this paper. This time reduction would be even of 1/30 if applied to a complex wing modeled with its upper and lower surfaces.

Multipole Expansion Approximation

As it appears from Eqs. (2) and (3), the coefficients of the system used by Morino's method to calculate the potential distribution require the evaluation of sources and doublets in an integral form. These integrals can be approximated by a multipole expansion as is done in Ref. 20 for sources and Ref. 1 for both sources and doublets and as is extensively explained in Ref. 11. This suggestion, which has not yet been introduced in Morino's method, can produce substantial savings, especially for calculations related to complex configurations. Nevertheless, its usefulness is not marginal, even in a single lifting surface analysis, because of the high cost of calculating the integrals related to the wake. In view of the extensive documentation available about this concept, it is not further explained here. Nevertheless, it is noted that the application of multipole expansions to the examples presented here has allowed the reduction of the computer time required for the calculation of source and doublet integrals of Eqs. (3) to one-quarter of that required for an exact calculation. This was done by using a conservative switching rule to determine the order of the multipole expansions to be used at varying distances from the sending panel.

Even if this saving is obtained only at a change in Mach number, since Morino's method does not require the evaluation of the coefficients for each reduced frequency, it is worthwhile in any case because it produces a substantial gain in this calculation without any need to resort to a specialized technique for the wake panels, such as that presented in Ref. 10.

Conclusions

This paper has shown how, by combining a lifting surface approximation, functional variable linking, and multipole expansions, the computational efficiency of Morino's method can be greatly improved without sacrificing its precision. This makes the method more and more usable for routine aeroelastic calculations. It is thought that this work will be helpful in making Morino's method a standard and unifying tool for aeroelastic analyses of both simple and complex configurations. This is irrespective of the existence of more efficient methods for particular configurations because, once a reasonable performance has been achieved in terms of computer time and cost, there surely will be a tendency to adopt a unifying analysis tool if a substantial saving can be achieved in documentation, training, and maintenance, as it has been demonstrated by the experience acquired in the use of the finite-element method for structural analysis.

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